MIDTERM REVIEW---HONORS

NAME: Key

REAL NUMBERS --- Section 1.1

1) Given the set: $\left\{-3, \sqrt{12}, \frac{16}{4}, -\sqrt{64}, 7.121121112..., 7\frac{2}{5}, -9.55, 0, 8, \frac{13}{72}, 4.1\overline{237}, .72\overline{3}\right\}$

List ALL the elements of the set that belong to each set below.

a) Natural Numbers (N) $\frac{16}{4}, 8$

b) Whole Numbers (W) $\frac{16}{4}, 0, 8$

c) Integers (I) $-3, \frac{16}{4}, -\sqrt{64}, 0, 8$

d) Rational Numbers (Q) $-3, \frac{16}{4}, -\sqrt{64}, 7\frac{2}{5}, -9.55, 0, 8, \frac{13}{72}, 4.1\overline{237}, .72\overline{3}$

e) Irrational Numbers (Ir) $\sqrt{12}, 7.121121112...$

f) Real Numbers (R) $-3, \sqrt{12}, \frac{16}{4}, -\sqrt{64}, 7.121121112..., 7\frac{2}{5}, -9.55, 0, 8, \frac{13}{72}, 4.1\overline{237}, .72\overline{3}$

2) Match each equation to the name of the property illustrated by the equation.

2	a) $9 + (\sqrt{5} + 0) = 9 + (0 + \sqrt{5})$	1)	Associative Property of Multiplication
1	b) $2+7\left(\frac{2}{3}\cdot\frac{3}{2}\right)=2+\left(7\cdot\frac{2}{3}\right)\cdot\frac{3}{2}$	2)	Commutative Property of Addition
5	c) $2 + 7 \cdot 1 = 2 + 7$	3)	Inverse Property of Multiplication
4	d) $6(-4 + 4) = 6(0)$	4)	Inverse Property of Addition
7	e) $6(-4 + 4) = -24 + 24$	5)	Identity Property of Multiplication
6	f) $9 + (\sqrt{5} + 0) = 9 + \sqrt{5}$	6)	Identity Property of Addition
3	g) $2+7\left(\frac{2}{3}\cdot\frac{3}{2}\right)=2+7\cdot 1$	7)	Distributive Property

3) Complete the following using the order of operations. Do only **ONE STEP** at a time.

$8 + (-3^2 + 3) \div 2 \cdot 4 - 6$

$8 + (-9 + 3) \div 2 \cdot 4 - 6$

$8 - 6 \div 2 \cdot 4 - 6$

$8 - 3 \cdot 4 - 6$

$8 - 12 - 6$

$-4 - 6$

$= -10$

ORDER AND ABSOLUTE VALUE --- Section 1.2

4) Evaluate the following expression given x = –3 and y = –5.

$$\frac{2|y| - x^3}{|y^2 + 3x|} = \frac{2|-5| - (-3)^3}{|(-5)^2 + 3(-3)|} = \frac{2 \cdot 5 - (-27)}{|25 - 9|} = \frac{10 + 27}{16} = \frac{37}{16}$$

5) Given $x > 3$, write each of the following without absolute value bars.

a. $|x^2 - 9| =$ $x^2 - 9$

b. $|9 - x^2| =$ $-(9 - x^2) = x^2 - 9$

c. $|3 - x| =$ $-(3 - x) = x - 3$

d. $|-x^2 - 2| =$ $-(-x^2 - 2) = x^2 + 2$

RATIONAL EXPONENTS --- Sections 1.3/1.6

6) Evaluate each of the following. Show the problem you use to complete each.

a. $(-5)^2 =$ $(-5)(-5) = 25$

b. $-5^2 =$ $-(5)(5) = -25$

7) Simplify (no like bases, no negative exponents) each of the following.

a. $\dfrac{(2k)^{-3}(k^{-5})^{-1}}{(6k^{-2})^{-1}(k^3)^{-6}} = \dfrac{k^5}{(2k)^3(6^1k^2)(k^{-18})}$

$\dfrac{6k^5}{8k^3 \cdot k^2 \cdot k^{-18}} = \boxed{\dfrac{3k^{18}}{4}}$

b) $\dfrac{3x^3y^{-\frac{1}{5}}z^{\frac{2}{9}}}{4x^{-2}z^{\frac{1}{3}}} = \boxed{\dfrac{3x^5}{4y^{\frac{1}{5}}z^{\frac{1}{9}}}}$

$\frac{2}{9} - \frac{1}{3} = \frac{-1}{9}$

$\frac{2}{9} - \frac{3}{9} = \frac{-1}{9}$

8) Simplify $243^{-\frac{2}{5}}$. $\left(\dfrac{1}{243}\right)^{2/5} = \left(\dfrac{1}{\sqrt[5]{35}}\right)^2 = \left(\dfrac{1}{3}\right)^2 = \boxed{\dfrac{1}{9}}$

RADICAL EXPRESSIONS --- Section 1.7

9) Simplify: $\sqrt[4]{x^{12}y^{46}z^{103}} = \boxed{x^3y^{11}z^{25}\sqrt{y^2z^3}}$

10) Add /Subtract: $-3\sqrt{54} - 2\sqrt{18} + 5\sqrt{96}$

$-3\sqrt{9 \cdot 6} - 2\sqrt{9 \cdot 2} + 5\sqrt{16 \cdot 6}$

$-9\sqrt{6} - 6\sqrt{2} + 20\sqrt{6}$

$\boxed{11\sqrt{6} - 6\sqrt{2}}$

11) Rationalize the denominator:

a) $\sqrt[3]{\frac{5}{24}} = \frac{\sqrt[3]{5}}{\sqrt[3]{24}} = \frac{\sqrt[3]{5}}{2\sqrt[3]{3}} \cdot \frac{\sqrt[3]{3^2}}{\sqrt[3]{3^2}} = \boxed{\frac{\sqrt[3]{45}}{6}}$

$24 = 2^3 \cdot 3$

b) $\frac{2+2\sqrt{6}}{4-\sqrt{6}} \cdot \frac{4+\sqrt{6}}{4+\sqrt{6}} = \frac{8+2\sqrt{6}+8\sqrt{6}+12}{16-6}$

$= \frac{20+10\sqrt{6}}{10} = \boxed{2+\sqrt{6}}$

COMPLEX NUMBERS --- Section 2.3

12) Multiply: $3\sqrt{-27} \cdot 2\sqrt{-60} = 3\sqrt{-1 \cdot 3^3} \cdot 2\sqrt{-1 \cdot 4 \cdot 15} = 90\sqrt{3} \cdot 4i\sqrt{15}$

$= 36i^2\sqrt{45} = \boxed{-108\sqrt{5}}$

13) Simplify: $i^{2306} = i^2 = \boxed{-1}$

14) Add/Subtract: $(7 - 3i) + (-4 + 7i) + (\bar{6} + 8i) = \boxed{-3 + 12i}$

15) Rationalize: $\frac{4+3i}{2-5i} \cdot \frac{2+5i}{2+5i} = \frac{8+20i+6i-15}{4+25} = \frac{-7+26i}{29} = \boxed{\frac{-7}{29} + \frac{26i}{29}}$

POLYNOMIALS --- Section 1.3

**16) Give the degree and the most specific classification for: $3x^3 + 5x^2y^4 - 8xyz$

Degree 6
trinomial
multi-variate

17) Add/Subtract: $(3x^2 - 2x + 3) + 4(5x^2 - 2x + 4) + (\bar{2}x^2 + \bar{6}x + 7)$

$(3x^2 - 2x + 3) + (20x^2 - 8x + 16) + (-2x^2 - 6x + 7)$

$\boxed{21x^2 - 16x + 26}$

18) Multiply: $(3x + 5)(2x^2 - 3x + 5)$

$6x^3 - 9x^2 + 15x$

$10x^2 - 15x + 25$

$\boxed{6x^3 + x^2 + 25}$

19) Divide: $(4x^4 - 4x^3 - 1) \div (2x^2 - 1)$

$2x^2 - 2x + 1$ R $-2x$

$$2x^2 - 2x + 1 - \frac{2x}{2x^2 - 1}$$

20) Square the trinomial: $(3x^3 - 7z^8 + 2)^2$

$(3x^3 - 7z^8 + 2)(3x^3 - 7z^8 + 2) =$

$9x^6 - 21x^3z^8 + 6x^3 - 21x^3z^8 + 49z^{16} - 14z^8 + 6x^3 - 14z^8 + 4$

$$9x^6 - 42x^3z^8 + 12x^3 + 49z^{16} - 28z^8 + 4$$

THE BINOMIAL THEOREM --- Sections 1.3 and 8.4

21) Use **Pascal's triangle** to expand $(4x + 3)^5$.

```
1                    0
1 1                  1
1 2 1                2
1 3 3 1              3
1 4 6 4 1            4
1 5 10 10 5 1        5
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$(4x)^5(3)^0 + 5(4x)^4(3)^1 + 10(4x)^3(3)^2 + 10(4x)^2(3)^3 + 5(4x)^1(3)^4 + (4x)^0(3)^5$

$$1024x^5 + 3840x^4 + 5760x^3 + 4320x^2 + 1620x + 243$$

COUNTING THEORY --- Section 8.6

22) Write the factorial notation and evaluate the following:

a) $P(10,7)$

$${}_{10}P_7 = \frac{10!}{(10-7)!} = \frac{10!}{3!} = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 604800$$

b) $C(10,7) = \frac{10!}{7!(10-7)!} = \frac{10!}{7!3!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120$

23) A teacher has 5 books

a) How many ways can she arrange 3 of these on her desk?

order matter $P(5,3) = \frac{5!}{(5-3)!} = 60$

b) How many ways can she pick 3 of these give to another teacher?

order does not matter $C(5,3) = \frac{5!}{3!(5-3)!} = \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = 10$

24) A math class consisting of 18 males and 12 females is preparing for the midterm exam.

a) How many study groups of 5 can be formed?

$C(30,5) = 142{,}506$ groups

b) How many study groups of 2 males and 3 females can be formed?

$C(18,2)$ AND $C(12,3) = 153 \cdot 220 = 33{,}660$ groups

FACTORING POLYNOMIALS --- Section 1.4

25) Factor each of the following polynomials COMPLETELY:

a) $27x^{27} - 64y^{15} = (3x^9 - 4y^5)(9x^{18} + 12x^9y^5 + 16y^{10})$

b) $20x^2 - 28x - 3 = (2x - 3)(10x + 1)$

-60, -30, 2

c) $25x^2 - 60x + 36 - 81y^2 = (5x-6)^2 - 81y^2 = [(5x-6) - 9y][(5x-6) + 9y] = (5x - 9y - 6)(5x + 9y - 6)$

d) $9x^2(2x^4 + 7)^3(x + 3)^7 - 2(2x^4 + 7)^4(5)(x + 3)^6$ $(2x^4+7)^3(x+3)^6(9x^3+27x^2-20x^4-70)$

$(2x^4+7)^3(x+3)^6[9x^2(x+3) - 10(2x^4+7)]$

$(9x^3+27x^2-20x^4-70)$

$(2x^4+7)^3(x+3)^6(-20x^4+9x^3+27x^2-70)$

$-(2x^4+7)^3(x+3)^6(20x^4-9x^3-27x^2+70)$

RATIONAL EXPRESSIONS --- Section 1.5

26) Reduce: $\dfrac{81-16x^2}{4x^2-x-18} = \dfrac{(9-4x)(9+4x)}{(4x-9)(x+2)} = \dfrac{-(9+4x)}{x+2}$

27) Divide: $\dfrac{x^2+4x+4}{3x+12} \div \dfrac{x+2}{x^2+4x} = \dfrac{(x+2)(x+2)}{3(x+4)} \cdot \dfrac{x(x+4)}{(x+2)} = \dfrac{x(x+2)}{3}$

28) Subtract: $\dfrac{x-7}{x^2+4x-5} - \dfrac{x-9}{x^2+3x-10} =$

$\dfrac{(x-7)}{(x+5)(x-1)}\cdot\dfrac{(x-2)}{(x-2)} - \dfrac{(x-9)}{(x+5)(x-2)}\cdot\dfrac{(x-1)}{(x-1)} =$

$\dfrac{x^2-9x+14}{(x+5)(x-1)(x-2)} - \dfrac{x^2-10x+9}{(x+5)(x-1)(x-2)} = \dfrac{x+5}{(x+5)(x-1)(x-2)} = \dfrac{1}{(x-1)(x-2)}$

29) Simplify: $\dfrac{\frac{1}{x-1} - \frac{6}{x}}{\frac{1}{x}} = \dfrac{\frac{x}{x(x-1)} + \frac{-6(x-1)}{x(x-1)}}{\frac{1}{x}} = \dfrac{\frac{x}{x(x-1)} + \frac{-6x+6}{x(x-1)}}{\frac{1}{x}} = \dfrac{\frac{-5x+6}{x(x-1)}}{\frac{1}{x}}$

$= \dfrac{-5x+6}{x(x-1)} \cdot \dfrac{x}{1} = \dfrac{-5x+6}{x-1}$

EQUATIONS/APPLICATIONS --- Sections 2.1, 2.2, 2.4, 2.5, 2.6

30) Solve the equation: $\frac{y+3}{y-6}+\frac{y-2}{y+2}=\frac{2y^2+9}{y^2-4y-12}$

$y \neq 6, y=-2$

$\frac{y+3}{y-6}+\frac{y-2}{y+2}=\frac{2y^2+9}{(y-6)(y+2)}$ $(y-6)(y+2)$

$(y+3)(y+2)+(y-2)(y-6)=2y^2+9$

$y^2+5y+6+y^2-8y+12=2y^2+9$

$-3y+18=9$

$-3y=-9$

$\boxed{y=3}$

31) Solve the equation $\frac{2b}{c-1}=b-d$; for c

$\frac{2b}{b-d}=c-1$ $\frac{2b}{b-d}+\frac{b-d}{b-d}=c$

$\frac{2b}{b-d}+1=c$ $\boxed{\frac{3b-d}{b-d}=c \quad b\neq d}$

32) Solve the equation: $5(x^2-1)=2x(x+6)$

a) Using the quadratic formula

$5x^2-5=2x^2+12x$

$3x^2-12x-5=0$

$a=3 \quad b=-12 \quad c=-5$

$x=\frac{12\pm\sqrt{144-4(3)(-5)}}{6}$ $\boxed{x=\frac{6\pm\sqrt{51}}{3}}$

$x=\frac{12\pm\sqrt{144+60}}{6}=\frac{12\pm\sqrt{204}}{6}=\frac{12\pm 2\sqrt{51}}{6}$

b) By completing the square

$3x^2-12x-5=0$

$3x^2-12x=5$

$x^2-4x=\frac{5}{3}$

$x^2-4x+4=\frac{5}{3}+\frac{12}{3}$

$(x-2)^2=\frac{17}{3}$

$x-2=\pm\sqrt{\frac{17}{3}}$ $x=2\pm\frac{\sqrt{51}}{3}$

$\boxed{x=\frac{6\pm\sqrt{51}}{3}}$

33) Calculate the discriminate and tell what kind and how many solutions each of the following quadratic equations has.

a) $4x^2-4x=3$ $4x^2-4x-3=0$ $a=4 \quad b=-4 \quad c=-3$ b^2-4ac $16-4(4)(-3)$ $16+48$ $\boxed{64}$ 2 Real Rational

b) $4x^2-12x=-9$ $4x^2-12x+9=0$ $a=4 \quad b=-12 \quad c=9$ b^2-4ac $144-4(4)(9)$ $144-144$ 0 1 Real Rational

c) $4x^2-4x=10$ $4x^2-4x-10=0$ $a=4 \quad b=-4 \quad c=-10$ b^2-4ac $(-4)^2-4(4)(-10)$ $16+160$ 166 2 Real Irrational

d) $4x^2-12x=-10$ $4x^2-12x+10=0$ $a=4 \quad b=-12 \quad c=10$ b^2-4ac $144-4(4)(10)$ $144-160$ -16 2 Non-Real Imaginary

34) Solve the equation: $\sqrt{3x+4}-1=\sqrt{x+5}$

$3x+4-2\sqrt{3x+4}+1=x+5$

$-2\sqrt{3x+4}=-2x$

$\sqrt{3x+4}=x$

$3x+4=x^2$

$0=x^2-3x-4$

$0=(x-4)(x+1)$

$x=4 \quad x=-1$

Check $\sqrt{16}-1=\sqrt{9}$ $4-1=3$ ✓

$\sqrt{1}-1=\sqrt{4}$ $0\neq 2$

$\{4\}$

35) Solve the equation: $2x^{-4} - 5x^{-2} = 3$

let $y = x^{-2}$

$2y^2 - 5y - 3 = 0$

$(2y + 1)(y - 3) = 0$

$y = -\frac{1}{2}$ $\quad y = 3$

$x^{-2} = -\frac{1}{2}$ $\quad x^{-2} = 3$

$x^2 = -2$ $\quad x^2 = \frac{1}{3}$

$x = \pm i\sqrt{2}$ $\quad x = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$

$\left\{\pm i\sqrt{2}, \pm\frac{\sqrt{3}}{3}\right\}$

36) If Andrew can paint the house in 8 hours and Pat can paint the house in 6 hours, how long will it take them to paint the house if they work together?

	Part/hr	time (hr)	Part Completed
Andrew	$\frac{1}{8}$	x	$\frac{x}{8}$
Pat	$\frac{1}{6}$	x	$\frac{x}{6}$

let x = time spent working together (hrs)

Andrew's Part + Pat's Part = whole

$\left(\frac{x}{8} + \frac{x}{6} = 1\right)48$

$6x + 8x = 48$

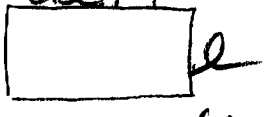
$14x = 48$

$x = 3\frac{6}{7}$

$\frac{6}{7} \cdot 60 = 51.\overline{42}$ ~~34.28~~ 25.71

Together it would take them 3 hrs ~~34~~ 26 min

37) The width of the rectangle is 9 inches longer than twice the length. If the area of the rectangle is 18 square inches, what are the dimensions of the rectangle?



let l = length of rectangle

let width = $2l + 9$

$A = (\text{length})(\text{width})$

$18 = (l)(2l+9)$

$18 = 2l^2 + 9l$

$0 = 2l^2 + 9l - 18$

$0 = (2l - 3)(l + 6)$

$l = 3/2$ $\quad l = -6$

The length is 1.5 inches
the width is 12 inches

38) Two friends decide to meet in Chicago to attend a Cub's baseball game. Rob travels 310 miles in the same time that Carl travels 295 miles. Rob's trip uses more interstate highways and he can average 3 mph more than Carl. What is Rob's average speed?

	Rate (mph)	time (hr)	Distance (miles)
Rob	x	$\frac{310}{x}$	310
Carl	$x-3$	$\frac{295}{x-3}$	295

$t = \frac{D}{r}$

let x = rate at which Rob travels (mph)

Time Rob Travels = Time Carl travels

$$\frac{310}{x} = \frac{295}{x-3}$$

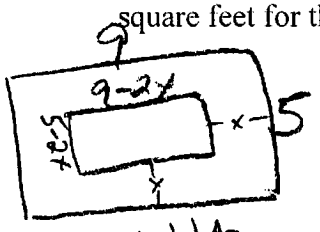
$310x - 930 = 295x$

$15x = 930$

$x = 62$

Rob's avg speed is 62 mph

39) A landscape architect has included a rectangular flower bed measuring 9 feet by 5 feet in her plans for a new building. She wants to use two colors of flowers in the bed, one in the center and the other for a border of the same width on all four sides. If she has enough plants to cover 24 square feet for the border, how wide can the border be?



let x = width of border (ft)

Area of border can be 24 ft²

Area of border = Area of entire garden − Area of center garden

$24 = (9)(5) - (9-2x)(5-2x)$

$24 = 45 - [45 - 28x + 4x^2]$

$24 = 45 - 45 + 28x - 4x^2$

$4x^2 - 28x + 24 = 0$

$x^2 - 7x + 6 = 0$

$(x-6)(x-1) = 0$

not feasible → $x \neq 6$ $x = 1$

The border should be 1 ft

40) Including an 8% sales tax, an inn charges $162 per night. Find the inn's nightly cost before the tax is added.

let x = $ cost of stay at Inn before tax (pretax)

Final charge = pre-tax + tax

$162 = x + .08x$

$162 = 1.08x$

$150 = x$

The pretax charge is $150